THE ASTROPHYSICAL JOURNAL, 469:166–170, 1996 September 20

METAL DEPENDENCES OF TWO CONVECTION THEORIES FOR COOL STELLAR ENVELOPES

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Received 1996 January 2; accepted 1996 April 2

ABSTRACT

Most theories of turbulent convection in stellar envelopes assume incompressible flow, and so require the assignment of a characteristic length scale from external evidence. In mixing-length theory, this length l is usually assigned to be a constant, α, times the local pressure scale height, H_P, or, alternatively, times the distance from the top of the convection zone, z. The new full-spectrum-of-turbulence theory of Canuto & Mazzitelli uses $l = z$, and therefore is formally parameter-free. Chieffi, Straniero, & Salaris have recently suggested that α in mixing-length theory depends on metallicity, Z, but they considered only low-mass stars. We do a similar analysis for stars of higher mass. Specifically, we compare predicted and observed effective temperatures of red giants and red supergiants of widely differing metallicities, but identical luminosities, within the mass range 5–10 $M_\odot$. The stars utilized belong to several open clusters in the Galaxy with $Z \approx 0.02$ and to the clusters NGC 330 and NGC 458 in the Small Magellanic Cloud with $Z = 0.002$–0.004. It appears that either α in mixing-length theory is independent of metallicity or, since the empirical effective temperatures of the SMC stars may have been underestimated, α increases slightly with decreasing metallicity. On the other hand, Canuto & Mazzitelli's theory with $l = z$ is found to perform quite well in all cases, within the possible errors of the observations and of the low-temperature opacities.

Subject headings: convection — open clusters and associations: general — stars: interiors — stars: late-type — supergiants — turbulence

1. INTRODUCTION

A requirement of any satisfactory theory of turbulent convection in stellar envelopes is the ability to reproduce the observed effective temperatures of the coolest stars over a wide range of masses, luminosities, and metallicities. Ideally, this should be accomplished without free parameters. The standard mixing-length theory (MLT), as presented for example by Böhm-Vitense (1958) and Cox & Giuli (1968), contains a number of free parameters, although only one—the average eddy mixing length—is ordinarily varied to achieve a fit between predicted and observed effective temperatures. Since the mixing length, l, is usually taken to be a constant, α_P, times the local pressure scale height, H_P, a consistency test amounts to determining whether α_P is a true constant.

For low-mass stars, including main-sequence stars and old red giants, no obvious contradiction has arisen to the assumption that α_P is independent of stellar mass (VandenBerg 1983, 1985, 1992; VandenBerg & Bridges 1984; Pedersen, VandenBerg, & Irwin 1990; Bergbusch & VandenBerg 1992; Chieffi, Straniero, & Salaris 1995). Among red giants and red supergiants of 3–10 $M_\odot$, however, the fitted values of α_P display a significant decline with increasing stellar mass, and equal the solar value (based on similar input physics) at $\sim 5\ M_\odot$ (Stothers & Chin 1995). Furthermore, metallicity also may affect α_P, principally through the molecular part of the total opacity, as seen in standard models of the Sun calculated with and without molecules (Sackmann, Boothroyd, & Fowler 1990; Kim, Demarque, & Guenther 1991; Guenther et al. 1992; Lydon, Fox, & Sofia 1993; Sackmann, Boothroyd, & Kraemer 1993; Chieffi et al. 1995). Although a larger molecular opacity (or, equivalently, a larger metallicity) always leads to an increase of the fitted value of α_P at a fixed effective temperature, stars of different metallicities are known to have different effective temperatures. In fact, stars with a high metal abundance are *cooler* than metal-weak stars. To determine whether α_P actually varies with metallicity requires, therefore, the use of very accurate molecular opacities. Using different sets of published opacities, VandenBerg and his coworkers have found no obvious variation of α_P among low-mass stars with widely varying initial metal abundances, $Z_e = 0.0001$–0.02, whereas Chieffi et al. (1995) and Salaris & Cassisi (1996) have inferred from similar data that α_P declines somewhat with decreasing metal abundance.

This state of affairs, at least regarding the strong mass dependence of α_P, indicates the need to tune α_P for each star, and so one naturally looks for an improvement or a replacement of mixing-length theory. Canuto & Mazzitelli (1991, 1992) have recently presented a new theory of stellar convection that includes the full size spectrum of turbulent eddies. This "full spectrum theory" (FST) is parameterized in terms of the mixing length of the largest eddy, which on the basis of laboratory evidence (assumed to be relevant to stars) Canuto & Mazzitelli (1991, 1992) take to be the distance, z, below the top of the convection zone. Generalizing slightly, they then allow l to be equal to a constant, α_z, times z, where α_z is of order unity. Their new model of convection, with $\alpha_z = 1$, leads to very good fits for the Sun, α Centauri A and B, other low-mass main-sequence stars, and red giants in the old open cluster M67 and in various globular clusters (Canuto & Mazzitelli 1991, 1992; D'Antona, Mazzitelli, & Gratton 1992; D'Antona & Mazzitelli 1994; Mazzitelli, D'Antona, & Caloi 1995; Fernandes & Neuforge 1995). Helioseismological observations of solar nonradial pulsation frequencies also strongly support the new theory (Paterno et al. 1993; Basu & Antia 1994a, 1994b; Baturin & Mironova 1995). Note that this theory is much better grounded conceptually and experimentally than MLT, as far as the spectrum of eddy sizes is concerned, and the

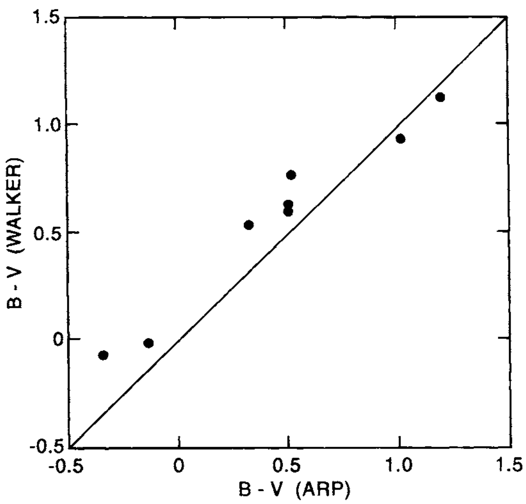

FIG. 1.—$B-V$ colors of stars in and around NGC 458, observed by Arp (1959) and by Walker (1987), for the range $V = 15$–18. This range of visual magnitude covers both the evolved supergiants and the stars near the tip of the main-sequence turnup.

theory is also parameter-free if one insists that $\alpha_z = 1$.

Since the testing and calibration of the MLT and FST models as a function of metallicity have been performed so far only for low-mass stars, and the reported results are still inconclusive, we have undertaken a theoretical investigation of red giants and red supergiants in the populous young clusters NGC 330 and NGC 458 belonging to the metal-poor Small Magellanic Cloud (SMC). Combining our new results with those previously published for red giants and red supergiants in young Galactic clusters of normal metallicity (Stothers & Chin 1995), we have derived the apparent dependences of α on metallicity for cool stars in the mass range 5–10 $M_\odot$ and in the initial metal abundance range $Z_e = 0.002$–0.02.

2. OBSERVATIONAL DATA

2.1. *NGC 330*

Effective temperatures of the cool supergiants in NGC 330 are now recognized to be more uncertain than they were at the time of our previous theoretical investigation of this cluster (Stothers & Chin 1992). At that time, the effective temperatures had been derived by Carney, Janes, & Flower (1985) from $J-K$ colors, and by us for a few stars from Robertson's (1974) $B-V$ colors, using a reddening of $E_{B-V} = 0.03$ based on the $B-V$ colors of normal-appearing blue supergiants. The $B-V$ colors themselves are not in doubt, having been confirmed many times since Robertson's study (Carney et al. 1985; Cayrel, Tarrab, & Richtler 1988; Balona 1992; Caloi et al. 1993; Bomans & Grebel 1994; Vallenari, Ortolani, & Chiosi 1994). With the same value of the reddening, Barbuy et al. (1991), Spite, Richtler, & Spite (1991), and Meliani, Barbuy, & Perrin (1995a) obtained in the same way very similar effective temperatures. The reddening, however, is somewhat uncertain, as the colors of the B-type giants on or near the main sequence imply $E_{B-V} = 0.12$ (Bessell 1991a), 0.09 (Caloi et al. 1993), or 0.07 (Bomans & Grebel 1994). Based on a larger reddening, the intrinsic colors of the cool supergiants become bluer and hence their inferred effective temperatures become hotter.

Higher effective temperatures have also been derived directly from the spectra of the cool supergiants, as reported by Bessell (1991b), Jasniewicz & Thévenin (1994), and Meliani et al. (1995a). These published values agree well among themselves, and so can be averaged for each star. In addition to collectively implying a substantial reddening, $E_{B-V} \approx 0.10$, they lead to a cluster metallicity of ~ 0.2 solar, which falls in line with most of the published abundance determinations for young SMC field stars (see, e.g., Luck & Lambert 1992; Thévenin & Jasniewicz 1992; Meliani, Barbuy, & Richtler 1995b; and references therein).

Previously, the inferred metallicity of NGC 330 had been ~ 0.1 solar (Grebel & Richtler 1992; and references therein). This is compatible with $E_{B-V} = 0.03$ and is still supported by the most recent Strömgren photometry (Hilker, Richtler, & Gieren 1995). Furthermore, Rolleston et al. (1993) now find an intermediate value of the metallicity, ~ 0.16 solar, for young SMC field stars.

In view of the persisting uncertainty, we shall adopt, again, a range of possible metallicities for NGC 330, specifically 0.1–0.2 solar, or $Z_e = 0.002$–0.004. The corresponding difference of inferred effective temperature for the coolest supergiants is 0.04 dex.

2.2. *NGC 458*

For NGC 458, the foreground Galactic reddening of $E_{B-V} = 0.03$ is taken to be the sole source of reddening in the case of this cluster near the edge of the SMC. Arp's (1959) photoelectrically calibrated photographic $B-V$ colors remain the only extensive color data available, except for nine cluster and field stars measured by Walker (1987) and 15 by Alvarado et al. (1995), both of whom made independent photoelectric observations. A comparison of Arp's and Walker's results for $V = 15$–18 is shown in Figure 1, revealing close agreement for the two coolest giants, which are of primary importance in this paper. Flower's (1977) relations between $B-V$ color and effective temperature are adopted as before; over the range needed, these relations are virtually identical to those of Böhm-Vitense (1981), who pointed out, in addition, their near-insensitivity to metallicity variations.

Little is known of the metal abundance of stars in NGC 458 except that it is low (Hagen & van den Bergh 1974). Therefore, we simply adopt a plausible metals range, $Z_e = 0.002$–0.004, as in our earlier study.

3. INPUT PHYSICS

In order to have as close a comparison as possible with our previously derived stellar models for $Z_e = 0.02$ (Stothers & Chin 1995), we have adopted the same basic input physics in our new evolutionary sequences for $Z_e = 0.002$ and 0.004. The only necessary changes were as follows. First, we assigned an initial helium abundance by mass of $Y_e = 0.24$ to reflect conditions in the SMC. Second, we adopted low-temperature ($T < 6000$ K) opacities, including diatomic and triatomic molecular sources, from Carson & Sharp (1995), while continuing to use the opacities of Iglesias, Rogers, & Wilson (1992) for higher temperatures up to $T = 10^8$ K, and the opacities of Cox & Stewart (1965, 1970) beyond.

The Carson & Sharp (1995) opacities form a supplement to Sharp's (1992) solar-abundance opacity tables (with some added improvements) for the case of reduced metal abundances $Z = 0$, 0.001, and 0.018. The corresponding helium abundances in the new tables are $Y = 0.250$, 0.216, and 0.212. Since, in general, low-temperature opacities are not very sensitive to Y, and the possible total errors of the opacities far exceed the minor differences arising from small variations of Y, we have simply assumed that the Carson & Sharp opacity tables are invariant to changes in Y and can be interpolated quadratically in Z. A comparison of their low-temperature opacities for $(Y, Z) = (0.212, 0.018)$ can be made with those derived independently by Alexander & Ferguson (1994) for $(Y, Z) = (0.280, 0.020)$. Differences are found to be small (average $|\Delta \log \kappa| = 0.08$) over the temperature range relevant to this paper, $\log T = 3.5$–3.8, but crop up in the same sense as the minor differences that we demonstrated previously between the Sharp (1992) and Alexander, Johnson, & Rypma (1983) opacities (Stothers & Chin 1993, Fig. 1). Low-temperature opacities computed independently by Kurucz (1992) and by Neuforge (1993) are very similar to each other, but differ somewhat from those due to Sharp (1992), though probably not by an amount that would be important for our present applications; Neuforge (1993) has graphically displayed the differences.

Other uncertainties in the physical input to the stellar models are known to have relatively little effect on the predicted band of the red giants and red supergiants in the H-R diagram (Stothers & Chin 1995, § 5), and so can be ignored. There is some uncertainty in defining the top of the convection zone (and also the effective temperature) of a cool star with an extended atmosphere (Mazzitelli et al. 1995), but for the purpose of this paper, standard assumptions are made.

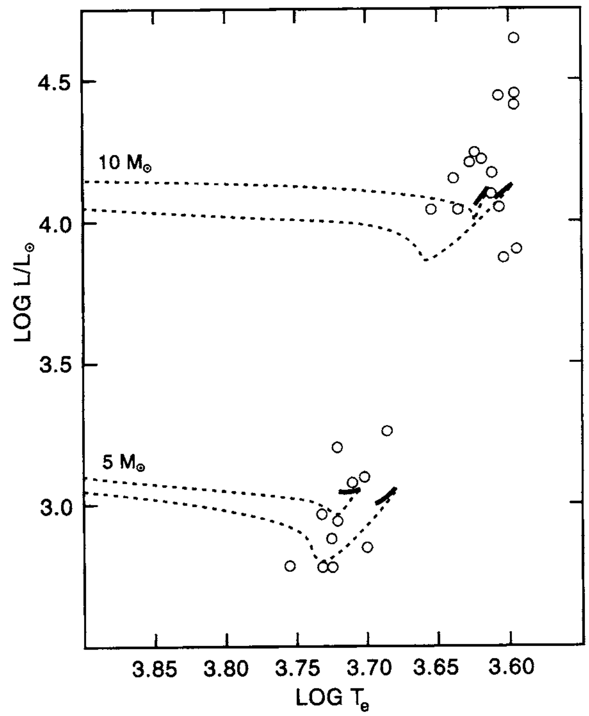

FIG. 2.—H-R diagram showing evolutionary tracks up to the second luminosity minimum on the red giant branch. Dashed segments represent very rapid stages. For each stellar mass, the hotter track refers to $Z_e = 0.002$ and the cooler track to $Z_e = 0.004$. Circles represent red giants in NGC 458 (faint clump) and red supergiants in NGC 330 (bright clump). The plotted effective temperatures for the observed stars are probably lower limits to the true values.

4. COMPARISON OF EVOLUTIONARY TRACKS WITH OBSERVATIONS

A preliminary attempt was made to fit the observed effective temperatures of cool evolved stars in NGC 458 and NGC 330 by using the α values that we had previously found by matching evolutionary tracks for $Z_e = 0.02$ to Galactic cluster H-R diagrams of similar age. Our new model results for 5 $M_\odot$ and 10 $M_\odot$ with low metallicities are presented in Table 1 and Figure 2. The slow evolution that occurs in the red giant region is depicted as a thick, continuous segment of the evolutionary track; it includes the first portion of core helium depletion. For a given mass and initial chemical composition, the tracks listed in Table 1 are almost indistinguishable from each other and so are shown only once.

Our initial pass at finding α was wholly successful in the case of NGC 458, where the typical red giant mass is ~ 5 $M_\odot$. Although the predicted difference of effective temperatures for the cases $Z_e = 0.002$ and 0.004 is $\Delta \log T_e = 0.02$, the observational scatter and other uncertainties are comparable to this difference. Notice that $\delta \log T_e / \delta \log$

TABLE 1

THEORETICAL RED GIANT BRANCHES

$M/M_\odot$	CONVECTION THEORY	l	DEEPEST q_{env}	RED TOP log $(L/L_\odot)$	RED TOP log T_e	SECOND RED BOTTOM log $(L/L_\odot)$	SECOND RED BOTTOM log T_e
			$Z_e = 0.002$				
5	FST	$1.0z$	0.669	3.06	3.70	3.05	3.71
10......	MLT	$1.8H_P$	0.474	4.12	3.63	4.11	3.63
	MLT	$2.2z$	0.471	4.12	3.61	4.11	3.62
	FST	$1.0z$	0.466	4.12	3.61	4.10	3.62
			$Z_e = 0.004$				
5	MLT	$2.1H_P$	0.434	3.06	3.68	3.00	3.69
	MLT	$2.0z$	0.417	3.06	3.67	3.00	3.69
	FST	$1.0z$	0.425	3.06	3.68	3.00	3.70
10......	MLT	$1.8H_P$	0.323	4.13	3.61	4.08	3.62
	MLT	$2.2z$	0.325	4.13	3.60	4.08	3.61
	FST	$1.0z$	0.333	4.13	3.60	4.08	3.60

$Z_e \approx -0.06$, which is an apparently general result applying at least over the range $Z_e = 0.002$–0.03 (see also Stothers & Chin 1995).

Matters are more complicated for NGC 330 because of the problematical reddening and hence the uncertain effective temperatures of its $\sim 10\ M_\odot$ red supergiants. We have formally plotted the "low" values of the effective temperatures, but note that the "high" values would be hotter by 0.04 dex. Our predictions for $Z_e = 0.002$ and 0.004 agree very well with the "low" values of effective temperature.

If, however, the "high" values should be more nearly correct, the consequent failure of the model predictions would imply the need for α values larger by roughly 35%, since, generally,

$$\delta \log T_e / \delta \log \alpha \approx 0.3$$

for both α_P and α_z (see also Stothers & Chin 1995). As a result, one would have to infer that α increases with decreasing metallicity. Nevertheless, this conclusion would not necessarily contradict the evidence from NGC 458, because we adopted Arp's colors (which are slightly redder than Walker's) as well as a minimal reddening of $E_{B-V} = 0.03$ for this cluster. Any additional reddening would lead to bluer intrinsic colors and hence hotter effective temperatures of the cluster's cool giants.

5. CONCLUSION

We tentatively conclude that, over the range $Z_e = 0.002$–0.02, there exists some observational evidence that α is essentially independent of metallicity, for both the MLT and FST models of convection. However, better observations of the effective temperatures of the red giants and red supergiants in NGC 458 and NGC 330 are badly needed. If the effective temperatures used for Figure 2 have been underestimated (and it is hard to see how they could have been *over*estimated), one would have to conclude that α increases slightly as the metallicity goes down.

This conclusion would be opposite to that reached by Chieffi et al. (1995) using MLT and the red giants (and, more uncertainly, the main-sequence stars) in globular clusters with $Z_e = 0.0001$–0.001. Their results, however, are formally consistent with *no* dependence of α_P on metallicity at the 1 σ level. It is only when a comparison with the Sun ($Z_e = 0.02$) is made that α_P appears to increase significantly with metallicity. Even then, the difference between $\alpha_P = 1.9$ (globular cluster red giants) and $\alpha_P = 2.2$ (Sun) amounts to only 15%, which corresponds to a shift in effective temperature of 0.01 dex. We note that the estimated errors of the observed effective temperatures of globular cluster red giants are at least that large (Frogel, Cohen, & Persson 1983). Moreover, the full range of α_P values derived for the Sun with different versions of the molecular opacities and other input physics is 1.9–2.2 (Kim et al. 1991; Guenther et al. 1992; Lydon et al. 1993; Sackmann et al. 1993).

Perhaps these two independent studies can be reconciled by recognizing that, within the possible errors affecting both, there is no clear contradiction to the supposition that α is independent of metallicity. Indeed, this simple assumption works remarkably well in predicting the observed effective temperatures of cool stars. Nevertheless, it must be kept in mind that the single parameter α is forced to subsume all the other assumptions and uncertainties present in the adopted convection theories.

Although the structure of the star's surface layers is affected very differently by the different choices of mixing length and of convection theory, the structure of the deeper layers is hardly altered. The nearly invariant depth of the convective envelope in terms of the mass fraction, $q_{\rm env}$, at the top of the red giant branch (Table 1) is just one illustration. For six of our evolutionary sequences listed in Table 1, we have computed the subsequent stages of evolution to the end of central helium burning. We also have available from Stothers & Chin (1992) four complete evolutionary sequences calculated with identical input physics, except for our earlier neglect of molecular sources in the low-temperature opacities and our earlier use of a preliminary version of the intermediate-temperature Livermore opacities (Rogers & Iglesias 1992). For a given stellar mass and initial chemical composition, the corresponding evolutionary sequences all exhibit very nearly the same blue loops on the H-R diagram.

This being the case, our previously published tracks for the stars in NGC 330 and NGC 458 retain their usefulness. In our earlier comparison of these tracks with the stars in NGC 458, however, Arp's measured $B-V$ colors for the blue and yellow giants were used (after dereddening) to obtain the giants' effective temperatures. According to Walker's data in Figure 1, these colors are possibly too blue by 0.19 mag, on the average. If so, the effective temperatures of the blue and yellow giants should be shifted to lower values by an amount approximately equal to $\Delta \log T_e = 0.7 (\log T_e - 3.8)$, where $\log T_e$ refers to the star's originally assigned effective temperature. The consequent reduction of the hottest observed effective temperatures by 0.2 dex in NGC 458 would produce much better agreement with our theoretical predictions for the tip of the helium-burning blue loop on the H-R diagram.

V. M. Canuto provided useful comments and the necessary inspiration for the present work, which was supported by the NASA Astrophysics and Climate Research Programs.

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